MIDAS/Gen & MIDAS/Civil 7.2.0.

Materiale “Strumas” per l’analisi delle murature

THEORETICAL BACKGROUND OF MASONRY MODEL

1. **Introduction**

Masonry, though a traditional material which has been used for construction for ages, is a complex material. It is a complex composite material and its mechanical behavior, which is influenced by a large number of factors, is not generally well understood. In engineering practice, many engineers have adopted an elastic analysis for the structural behavior of masonry using rather arbitrary elastic parameters and strengths of masonry. Such analyses can give wrong and misleading results. The proper way to obtain elastic parameters of masonry is through a procedure of homogenization described in the next section.

2. **Homogenization Techniques in Masonry Structures**

Masonry structures can be numerically analyzed if an accurate stress-strain relationship is employed for each constituent material and each constituent material is then separated individually. However, a three-dimension-analysis of a masonry structure involving even a very simple geometry would require a large number of elements and the nonlinear analysis of the structure would certainly be intractable. To overcome this computational difficulty, the orthotropic material properties proposed by Pande et al.[1],[2] can be introduced to model the masonry structure in the sense of an equivalent homogenized material. The equivalent material properties introduced in Pande et al. are based on a strain energy concept. The details of the procedure to obtain equivalent elastic parameters based on the homogenization technique are given in the following. The basic assumptions made to derive the equivalent material properties through the strain energy considerations are:

1. Brick and mortar are perfectly bonded
2. Head or perpend mortar joints are assumed to be continuous

[1] G. N. Pande, B. Kralj, and J. Middleton. Analysis of the compressive strength of masonry given by the equation $f_k = K\left(f_b'\right)^{\alpha}\left(f_m\right)^{\beta}$. The Structural Engineer, 71:7-12, 1994.

[2] G. N. Pande, J. X. Liang, and J. Middleton. Equivalent elastic moduli for brick masonry. Comp. & Geotech., 8:243-265, 1989.

The second assumption is necessary in the homogenization procedure and it has been shown[3] that the assumption of continuous head joints instead of staggered joints, as they appear in practice, does not have any significant effect on the stress states of the constituent materials.

Let the orthotropic material properties of the masonry panel be denoted by $\bar{E}_x$, $\bar{E}_y$, $\bar{E}_z$, $\bar{\nu}_{xy}$, $\bar{\nu}_{xz}$, $\bar{\nu}_{yz}$, $\bar{G}_{xy}$, $\bar{G}_{yz}$, $\bar{G}_{xz}$, Fig. 1. The stress/strain relationship of the homogenized masonry material is represented by

$$\bar{\sigma} = \left[\bar{D}\right]\bar{\varepsilon} \tag{1}$$

or

$$\bar{\varepsilon} = \left[\bar{C}\right]\bar{\sigma} \tag{2}$$

where,

$$\bar{\sigma} = \left\{\bar{\sigma}_{xx}\bar{\sigma}_{yy}\bar{\sigma}_{zz}\bar{\tau}_{xy}\bar{\tau}_{yz}\bar{\tau}_{xz}\right\}^T$$

$$\bar{\varepsilon} = \left\{\bar{\varepsilon}_{xx}, \bar{\varepsilon}_{yy}, \bar{\varepsilon}_{zz}, \bar{\gamma}_{xy}, \bar{\gamma}_{yz}, \bar{\gamma}_{xz}\right\}^T \tag{3}$$

are the vectors of stresses and strains in the Cartesian coordinate system.

$$\left[\bar{C}\right] = \begin{bmatrix} \frac{1}{\bar{E}_x} & -\frac{\bar{\nu}_{xy}}{\bar{E}_x} & -\frac{\bar{\nu}_{xz}}{\bar{E}_x} & 0 & 0 & 0 \\ -\frac{\bar{\nu}_{yx}}{\bar{E}_y} & \frac{1}{\bar{E}_y} & -\frac{\bar{\nu}_{yz}}{\bar{E}_y} & 0 & 0 & 0 \\ -\frac{\bar{\nu}_{zx}}{\bar{E}_z} & -\frac{\bar{\nu}_{zy}}{\bar{E}_z} & \frac{1}{\bar{E}_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\bar{G}_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\bar{G}_{yz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\bar{G}_{xz}} \end{bmatrix} \tag{4}$$

The details of the derivation of orthotropic elastic material properties of masonry in terms of the properties of the constituents, are given in Appendix I. In the mathematical theory of homogenization, there has been an issue relating to the sequence of homogenization, if there are more than two constituents. For example, if we homogenize bricks and mortar in head joints first and then homogenize the resulting material with bed joints at the second stage, then result may not be the same if we had followed a different sequence. However, it has been shown in the case of masonry, the sequence of homogenization does not have any significant influence. Here we present in Appendix I equations for equivalent properties if bricks and bed joints are homogenized first. It is noted that, in Pande et al., the equivalent material properties were derived with the brick and the head mortar joint being homogenized first. The equivalent orthotropic material properties derived from the homogenization procedure are used to construct the stiffness matrix in the finite element

[3] R. Luciano and E. Sacco. A damage model for masonry structures. Eur. J. Mech., A/Solids, 17:285-303,1998.

analysis procedure and, from this equivalent stress/strains are then calculated. The stresses/strains in the constituent materials can be evaluated through structural relationships, i.e.,

$$\sigma_b = [S_b]\bar{\sigma}$$
$$\sigma_{bj} = [S_{bj}]\bar{\sigma}$$
$$\sigma_{hj} = [S_{hj}]\bar{\sigma} \quad (5)$$

,where subscripts b, bj and hj represent brick, bed joint ad head joint, respectively. The structural relationships for strains can similarly be established. The structural matrices S are listed in Appendix II. From the results listed in Pande et al., it can shown that the orthotropic material properties are functions of

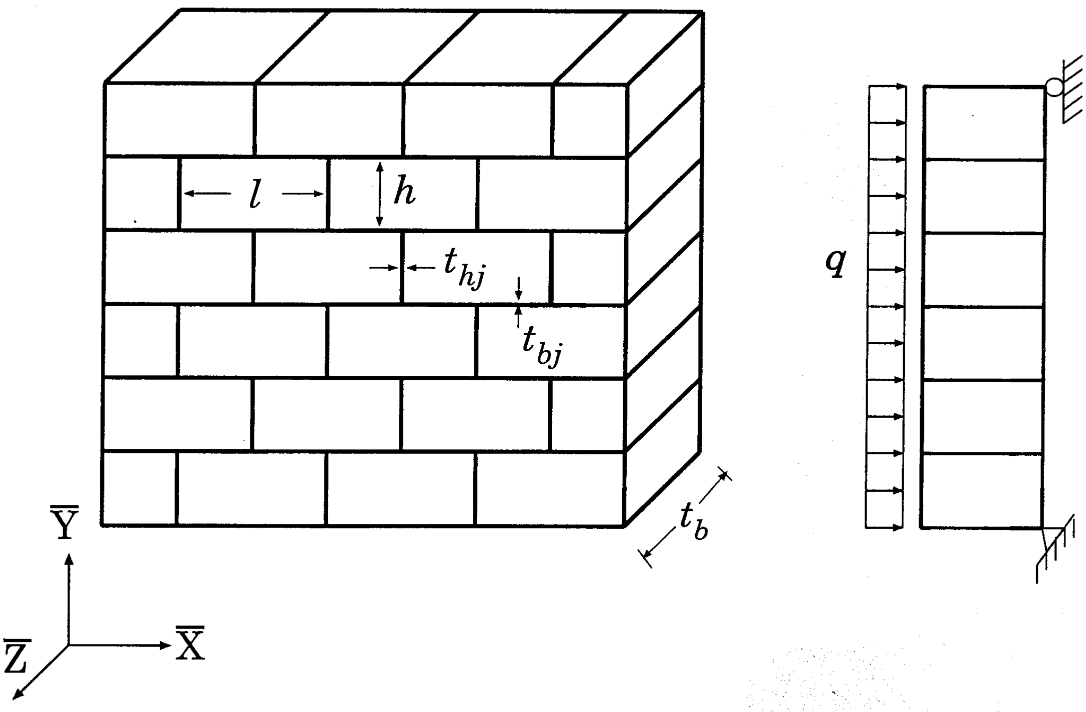

Figure1: Coordinate System used in Masonry Panel

1. Dimensions of the brick, length, height and width
2. Young's modulus and Poisson's ratio of the brick material
3. Young's modulus and Poisson's ratio of the mortar in the head and bed joints
4. Thickness of the head and bed mortar joints

3. Criteria for Failure for Constituents

Failure of masonry can be based on the micromechanical behaviour. Here, once the equivalent stresses/strains in the masonry structure are calculated, stresses/strains of the constituent materials can be derived based on the structural relationship in eq. (5). The failure criterion for each constituents, is next applied and the local failure mode can then be evaluated, for detail procedure see Lee et al. [4]

Although there are a number of criteria for the masonry model such as Mohr-Coulomb and so on, the masonry model in MIDAS currently determines the tensile failure referring only to the user-input tensile strength. After the tensile cracks occur, the masonry model is assumed to have elastic stiffness and the crack positions can be traced by post processor of solid stresses. This assumption is regarded to be enough to find the crack initiating positions in the structure and to understand general crack propagation. Also this assumption contributes to the robust convergence of the analysis. More advanced failure criteria are developed in the near future based on the abundant research.

Once cracking occurs in any constituent material, the effect is smeared onto the neighboring equivalent orthotropic material through another homogenization technique which has been previously applied to masonry in two-dimensional cases. A considerable amount of research and development has taken place in this area. A technique of 'critical plane approach' has been developed and applied to a project in Canada. However, this aspect is beyond the scope of the current capability.

---

[4] J. S. Lee, G. N. Pande, J. Middleton, and B. Kralj, Numerical modeling of brick masonry panels subject to lateral loadings. Com. & Str., 61:735-745, 1996.

APPENDIX I
ORTHOTROPIC PROPERTIES OF MASONRY BASED ON STRAIN ENERGY RULE

Orthotropic material properties of masonry can be derived employing a strain energy concept and the details are given in the following. It is noted that homogenization is performed between brick and bed joint first. Similar details can also be obtained when brick and head joint are homogenized first.

Referring to Fig. 1, volume fraction of brick and bed joint can be described as

$$\mu_b = \frac{h}{h+t_{bj}}; \qquad \mu_{bj} = \frac{t_{bj}}{h+t_{bj}} \tag{A.1}$$

where subscript b and bj represent the brick and bed joint, respectively. If the brick and bed joint are homogenized in the beginning, the following stress/strain components in the sense of volume averaging can be established;

$$\begin{aligned} \sigma &= \left\{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{yz}, \tau_{zx}\right\}^T \\ \varepsilon &= \left\{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}\right\}^T \end{aligned} \tag{A.2}$$

where,

$$\sigma_{xx} = \frac{1}{V}\sum_{i=1}^{2}\int_{Vi} \sigma_{xxi} dV_i \tag{A.3}$$

$$\varepsilon_{xx} = \frac{1}{V}\sum_{i=1}^{2}\int_{Vi} \varepsilon_{xxi} dV_i \tag{A.4}$$

and I=1 for brick, I=2 for bed joint. For each strain component,

$$\begin{aligned} \varepsilon_{xxi} &= \frac{1}{E_i}\left(\sigma_{xxi} - \nu_i \sigma_{yyi} - \nu_i \sigma_{zzi}\right) \\ \varepsilon_{yyi} &= \frac{1}{E_i}\left(\sigma_{yyi} - \nu_i \sigma_{xxi} - \nu_i \sigma_{zzi}\right) \\ \varepsilon_{zzi} &= \frac{1}{E_i}\left(\sigma_{zzi} - \nu_i \sigma_{xxi} - \nu_i \sigma_{yyi}\right) \\ \gamma_{xyi} &= \frac{\tau_{xyi}}{G_{xyi}} \\ \gamma_{yzi} &= \frac{\tau_{yzi}}{G_{yzi}} \\ \gamma_{xzi} &= \frac{\tau_{xzi}}{G_{xzi}} \end{aligned} \tag{A.5}$$

Now the strain energy for each component and 1 layer prism can be denoted as

$$\begin{aligned} U_{re} &= \sum_{i=1}^{2}\frac{1}{2}\int_{Vi}\left(\sigma_{xxi}\varepsilon_{xxi} + \sigma_{yyi}\varepsilon_{yyi} + \sigma_{zzi}\varepsilon_{zzi} + \tau_{xyi}\gamma_{xyi} + \tau_{yzi}\gamma_{yzi} + \tau_{xzi}\gamma_{xzi}\right)dV_i \\ U_e &= \frac{1}{2}\int_{V}\left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{zz}\varepsilon_{zz} + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz}\right)dV \end{aligned} \tag{A6}$$

where re and e represent the component and layer prism, respectively, and it is obvious that

$$U_{re} = U_e \qquad \text{A7}$$

Introduce auxiliary stresses/strains,

$$\begin{aligned}
\sigma_{xxi} &= \sigma_{xx} + A_{xxi} \\
\sigma_{yyi} &= \sigma_{yy} \\
\sigma_{zzi} &= \sigma_{zz} + A_{zzi} \\
\tau_{xyi} &= \tau_{xy} \\
\tau_{yzi} &= \tau_{yz} \\
\tau_{xzi} &= \tau_{xz} + A_{xzi}
\end{aligned} \qquad \text{A.8}$$

and

$$\begin{aligned}
\varepsilon_{xxi} &= \varepsilon_{xx} \\
\varepsilon_{yyi} &= \varepsilon_{yy} + B_{yyi} \\
\varepsilon_{zzi} &= \varepsilon_{zz} \\
\gamma_{xyi} &= \gamma_{xy} + B_{xyi} \\
\gamma_{yzi} &= \gamma_{yz} + B_{yzi} \\
\gamma_{xzi} &= \gamma_{xz}
\end{aligned} \qquad \text{A.9}$$

then, from eqs. (A.3) & (A.8),

$$\begin{aligned}
\sum_{i=1}^{2} \mu_i A_{xxi} &= 0 \\
\sum_{i=1}^{2} \mu_i A_{zzi} &= 0 \\
\sum_{i=1}^{2} \mu_i A_{xzi} &= 0
\end{aligned} \qquad \text{A.10}$$

and

$$\begin{aligned}
\sum_{i=1}^{2} \mu_i B_{yyi} &= 0 \\
\sum_{i=1}^{2} \mu_i B_{xyi} &= 0 \\
\sum_{i=1}^{2} \mu_i B_{yzi} &= 0
\end{aligned} \qquad \text{A.11}$$

where, μ_1 and μ_2 represent the volume fraction of brick and bed joint, respectively.

From eqs. (A.5),(A.9) & (A.11),

$$
\begin{aligned}
E_x &= \alpha - \frac{\zeta^2}{\alpha} \\
\frac{1}{E_y} &= \frac{\mu_b}{E_b} + \frac{\mu_{bj}}{E_{bj}} + 2\chi_b\left(\frac{\nu_{zy}}{E_z} - \frac{\nu_b}{E_b}\right) + 2\chi_{bj}\left(\frac{\nu_{zy}}{E_z} - \frac{\nu_{bj}}{E_{bj}}\right) \\
E_z &= \alpha - \frac{\zeta^2}{\alpha} \\
\frac{1}{G_{xy}} &= \sum_i \frac{\mu_i}{G_{xyi}} \\
\frac{1}{G_{yz}} &= \sum_i \frac{\mu_i}{G_{yzi}} \\
G_{xz} &= \sum_i \left(\mu\, G_{xzi}\right) \\
\nu_{xy} &= \chi - \frac{\chi\zeta}{\alpha} \\
\nu_{xz} &= \frac{\zeta}{\alpha} \\
\nu_{zy} &= \chi - \frac{\chi\zeta}{\alpha}
\end{aligned}
\qquad \text{A.12}
$$

where,

$$\alpha = \frac{\mu_b E_b \left(1-\nu_{bj}^2\right) + \mu_{bj} E_{bj} \left(1-\nu_b^2\right)}{\left(1-\nu_b^2\right)\left(1-\nu_{bj}^2\right)}$$

$$\zeta = \frac{\mu_b \nu_b E_b \left(1-\nu_{bj}^2\right) + \mu_{bj} \nu_{bj} E_{bj} \left(1-\nu_b^2\right)}{\left(1-\nu_b^2\right)\left(1-\nu_{bj}^2\right)}$$

$$\chi_b = \frac{\mu_b \nu_b}{1-\nu_b} \qquad \text{A.13}$$

$$\chi_{bj} = \frac{\mu_{bj} \nu_{bj}}{1-\nu_{bj}}$$

$$\chi = \chi_b + \chi_{bj}$$

and the relationship below can also be established;

$$\nu_{yx} = \nu_{xy} \frac{E_y}{E_x} \qquad \text{A.14}$$

For the system of masonry panel, the homogenization is applied to the layered material and head joint based on the assumption of continuous head joint. Now, volume fractions of the constituent materials are

$$\mu_{eq} = \frac{l}{l+t_{hj}}; \qquad \mu_{hj} = \frac{t_{hj}}{l+t_{hj}} \qquad \text{A.15}$$

where, subscript eq and hj represent layered material and head joint, respectively. As in the previous case, the following stress/strain components in the sense of volume averaging can be established;

$$\bar{\sigma} = \left\{\bar{\sigma}_{xx}, \bar{\sigma}_{yy}, \bar{\sigma}_{zz}, \bar{\tau}_{xy}, \bar{\tau}_{yz}, \bar{\tau}_{zx}\right\}^T$$

$$\bar{\varepsilon} = \left\{\bar{\varepsilon}_{xx}, \bar{\varepsilon}_{yy}, \bar{\varepsilon}_{zz}, \bar{\gamma}_{xy}, \bar{\gamma}_{yz}, \bar{\gamma}_{zx}\right\}^T \qquad \text{A.16}$$

Introducing auxiliary stresses/strains,

$$\begin{aligned}
\bar{\sigma}_{xxi} &= \bar{\sigma}_{xx} \\
\bar{\sigma}_{yyi} &= \bar{\sigma}_{yy} + C_{yyi} \\
\bar{\sigma}_{zzi} &= \bar{\sigma}_{zz} + C_{zzi} \\
\bar{\tau}_{xyi} &= \bar{\tau}_{xy} \\
\bar{\tau}_{yzi} &= \bar{\tau}_{yz} + C_{yzi} \\
\bar{\tau}_{xzi} &= \bar{\tau}_{xz}
\end{aligned} \qquad \text{A.17}$$

and

$$\begin{aligned}
\bar{\varepsilon}_{xxi} &= \bar{\varepsilon}_{xx} + D_{xxi} \\
\bar{\varepsilon}_{yyi} &= \bar{\varepsilon}_{yy} \\
\bar{\varepsilon}_{zzi} &= \bar{\varepsilon}_{zz} \\
\bar{\gamma}_{xyi} &= \bar{\gamma}_{xy} + D_{xyi} \\
\bar{\gamma}_{yzi} &= \bar{\gamma}_{yz} \\
\bar{\gamma}_{xzi} &= \bar{\gamma}_{xz} + D_{xzi}
\end{aligned} \qquad \text{A.18}$$

where, i=1 & i=2 represent the layered material and head joint, respectively. Following the same procedure and defining the following coefficients,

$$\begin{aligned}
\bar{\alpha} &= \frac{\mu_{eq} E_y}{1-\nu_{yz}\nu_{zy}} + \frac{\mu_{hj} E_{hj}}{1-\nu_{hj}^2} \\
\bar{\beta} &= \frac{\mu_{eq} E_z}{1-\nu_{yz}\nu_{zy}} + \frac{\mu_{hj} E_{hj}}{1-\nu_{hj}^2} \\
\bar{\zeta} &= \frac{\mu_{eq} \nu_{yz} E_z}{1-\nu_{yz}\nu_{zy}} + \frac{\mu_{hj} \nu_{hj} E_{hj}}{1-\nu_{hj}^2} \\
\bar{\chi}_{eq} &= \frac{\mu_{eq}\left(\nu_{zx} + \nu_{yx}\nu_{zy}\right)}{1-\nu_{yz}\mu_{zy}} \\
\bar{\chi}_{hj} &= \frac{\mu_{hj}\nu_{hj}}{1-\nu_{hj}} \\
\bar{\chi} &= \bar{\chi}_{eq} + \bar{\chi}_{hj} \\
\bar{\lambda}_{eq} &= \frac{\mu_{eq}\left(\nu_{yx} + \nu_{yz}\nu_{zx}\right)}{1-\nu_{yz}\nu_{zy}} \\
\bar{\lambda}_{hj} &= \frac{\mu_{hj}\nu_{hj}}{1-\nu_{hj}} \\
\bar{\lambda} &= \bar{\lambda}_{eq} + \bar{\lambda}_{hj}
\end{aligned} \qquad \text{A.19}$$

the orthotropic material properties of the masonry panel are finally derived;

$$\frac{1}{\bar{E}_x} = \frac{\mu_{eq}}{E_x} + \frac{\mu_{hj}}{E_{hj}} + \bar{\lambda}_{eq}\left(\frac{\bar{\nu}_{yx}}{\bar{E}_y} - \frac{\nu_{xy}}{E_x}\right) + \bar{\lambda}_{hj}\left(\frac{\bar{\nu}_{yx}}{\bar{E}_y} - \frac{\nu_{hj}}{E_{hj}}\right) +$$
$$\bar{\chi}_{eq}\left(\frac{\bar{\nu}_{zx}}{\bar{E}_z} - \frac{\nu_{xz}}{E_x}\right) + \bar{\chi}_{hj}\left(\frac{\bar{\nu}_{zx}}{\bar{E}_z} - \frac{\nu_{hj}}{E_{hj}}\right)$$
$$\bar{E}_y = \frac{\bar{\alpha}\bar{\beta} - \bar{\zeta}^2}{\bar{\beta}}$$
$$\bar{E}_z = \frac{\bar{\alpha}\bar{\beta} - \bar{\zeta}^2}{\bar{\alpha}}$$
$$\frac{1}{\bar{G}_{xy}} = \frac{\mu_{eq}}{G_{xy}} + \frac{\mu_{hj}}{G_{hj}}$$
$$\bar{G}_{yz} = \mu_{eq} G_{yz} + \mu_{hj} G_{hj}$$
$$\frac{1}{\bar{G}_{xz}} = \frac{\mu_{eq}}{G_{xz}} + \frac{\mu_{hj}}{G_{hj}}$$
$$\bar{\nu}_{yx} = \bar{\lambda} - \frac{\overline{\zeta\chi}}{\bar{\beta}}$$
$$\bar{\nu}_{yz} = \frac{\bar{\zeta}}{\bar{\beta}}$$
$$\bar{\nu}_{zx} = \bar{\chi} - \frac{\overline{\zeta\lambda}}{\bar{\alpha}} \qquad \text{A.20}$$
$$\bar{\nu}_{zy} = \frac{\bar{\zeta}}{\bar{\alpha}}$$

Appendix II : STRUCTURAL RELATIONSHIP OF MASONRY

Structural relationship of each constituent materials with respect to overall masonry can be established through utilizing auxiliary stress/strain components introduced in Appendix I. Details of each relationship are now deduced.

As in eq. (5), the structural matrix has the following form;

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \qquad \text{A.21}$$

Solving the auxiliary stress/strain components in eqs. (A.17) & (A.18),

$$\begin{aligned} \bar{\sigma}_{yy,hj} &= \bar{\sigma}_{yy} + C_{yy,hj} \\ &= \frac{1}{1-\nu_{hj}\nu_{hj}}\left\{E_{hj}\bar{\varepsilon}_{yy} + \nu_{hj}E_{hj}\bar{\varepsilon}_{zz} + \left(\nu_{hj}+\nu_{hj}^2\right)\bar{\sigma}_{xx}\right\} \qquad \text{A.22} \\ &= \bar{\sigma}_{xx}\left\{\frac{\nu_{hj}}{1-\nu_{hj}} - \eta\left(\frac{\bar{\nu}_{yx}}{\bar{E}_y} + \frac{\nu_{hj}\bar{\nu}_{zx}}{\bar{E}_z}\right)\right\} + \bar{\sigma}_{yy}\left\{\eta\left(\frac{1}{\bar{E}_y} - \frac{\nu_{hj}\bar{\nu}_{zy}}{\bar{E}_z}\right)\right\} + \bar{\sigma}_{zz}\left\{\eta\left(\frac{\nu_{hj}}{\bar{E}_z} - \frac{\nu_{yz}}{\bar{E}_y}\right)\right\} \end{aligned}$$

where,

$$\eta = \frac{E_{hj}}{1-\nu_{hj}^2} \qquad \text{A.23}$$

Therefore,

$$\begin{aligned} S_{hj,21} &= \frac{\nu_{hj}}{1-\nu_{hj}^2} - \eta\left(\frac{\bar{\nu}_{xy}}{\bar{E}_y} + \frac{\nu_{hj}\bar{\nu}_{zx}}{\bar{E}_z}\right) \\ S_{hj,22} &= \eta\left(\frac{1}{\bar{E}_y} - \frac{\nu_{hj}\bar{\nu}_{zy}}{\bar{E}_z}\right) \qquad \text{A.24} \\ S_{hj,23} &= \eta\left(\frac{\nu_{hj}}{\bar{E}_z} - \frac{\bar{\nu}_{yz}}{\bar{E}_y}\right) \end{aligned}$$

The above equation can be rewritten as follows:

$$S_{hj,21} = \frac{\nu_{hj}}{1-\nu_{hj}} - \bar{\eta}_{hj}\left(\frac{\bar{\nu}_{yx}}{\bar{E}_y} + \frac{\nu_{hj}\bar{\nu}_{zx}}{\bar{E}_z}\right)$$

$$S_{hj,22} = \bar{\eta}_{hj}\left(\frac{1}{\bar{E}_y} - \frac{\nu_{hj}\bar{\nu}_{zy}}{\bar{E}_z}\right) \qquad \text{A.25}$$

$$S_{hj,23} = \bar{\eta}_{hj}\left(\frac{\nu_{hj}}{\bar{E}_z} - \frac{\bar{\nu}_{yz}}{\bar{E}_y}\right)$$

where,

$$\bar{\eta}_{hj} = \frac{E_{hj}}{1-\nu_{hj}^2} \qquad \text{A.26}$$

Using same procedure, the remaining non-zero coefficients can also be derived;

$$S_{hj,11} = 1.0$$

$$S_{hj,31} = \frac{\nu_{hj}}{1-\nu_{hj}} - \bar{\eta}_{hj}\left\{\frac{\nu_{hj}\bar{\nu}_{yx}}{\bar{E}_y} + \frac{\bar{\nu}_{zx}}{\bar{E}_z}\right\}$$

$$S_{hj,32} = \bar{\eta}_{hj}\left\{\frac{\nu_{hj}}{\bar{E}_y} - \frac{\bar{\nu}_{zx}}{\bar{E}_z}\right\}$$

$$S_{hj,33} = \bar{\eta}_{hj}\left\{\frac{1}{\bar{E}_z} - \frac{\nu_{hj}\bar{\nu}_{yz}}{\bar{E}_y}\right\} \qquad \text{A.27}$$

$$S_{hj,44} = 1.0$$

$$S_{hj,55} = \frac{G_{hj}}{\bar{G}_{yz}}$$

$$S_{hj,66} = 1.0$$

Solving for the unknowns A, B, C and D in eqs. (A.8), (A.9), (A.17) and (A.18), the structural matrix for each component can be derived and the full details will be omitted.