Guidelines for proper use of Plate elements

In structural analysis using finite element method, the analysis model is created by dividing the entire structure into finite elements. This procedure is known as finite element discretization. We then establish the relation between applied load and displacement for the structure defined by the assemblage of elements. This means that the stiffness of the overall structure thus depends on the accuracy of the stiffness of individual finite elements.

In the case of beam element widely used in analysis practice, the element formulation is based on theoretically exact equations, thereby resulting in an exact solution. However, in the case of other elements (2-dimensional and 3-dimensional elements) some errors are included due to adopted assumptions in theoretical formulation.

Many types of plate elements used to model two-dimensional structures such as slabs and walls have been developed. However, none of the plate elements provide the exact solution for all cases. In the majority of cases, adequate assumptions should be adopted to approach the exact solution.

The statement of “no plate elements provide the exact solution” means that no such a plate element currently exists, which gives acceptable solutions for all types of loads, boundary conditions and material properties even for two-dimensional plane domains. It also means that if the configuration of the plate element deviates from the shape assumed in the formulation of the element stiffness, the margin of error increases. In the case of two or three-dimensional structures, it is necessary to analyze the structural model so as to minimize the discretization error.

The following outlines some guidelines (practical considerations) to avoid situations, which cause errors in application of plate elements and to minimize possible errors:

■ Performance of a Plate Element

The relation between element load and nodal displacement is defined by the element stiffness matrix. The element stiffness matrix of each element used in structure discretization is assembled in global stiffness matrix defining the equilibrium equation of the entire structure. The equilibrium equation retains the following form:

$$[\mathbf{K}]\{\mathbf{u}\}=\{\mathbf{F}\}$$

where,

$[\mathbf{K}]$ = global stiffness matrix
$\{\mathbf{u}\}$ = vectors of unknown nodal displacements
$\{\mathbf{F}\}$ = vectors of applied nodal loads

Once we obtain the vectors of nodal displacements from the above equation, strains and stresses at any point in the structure are approximated through interpolation.

From the above consideration, we can summarize the performance criteria for plate elements in the following two points:

- How well does the element stiffness perform?
- How well does it interpolate the strains and stresses at any point?

The above two criteria are related to the element formulation, which are directly processed by the analysis program. However, the element type and shape selected by the user can seriously affect the accuracy of analysis results. Therefore, the guidelines will explain the selection of appropriate elements from the several plate elements provided by the program. We will then examine the major shape factors affecting the efficiency of the elements. Lastly, main considerations for using the plate elements are described.

■ Selection of Plate Elements

The plate element is an element whose thickness is much smaller than its length and effectively resists the applied loads by the combination of in-plane (membrane) stiffness and out-of-plane (bending) stiffness.

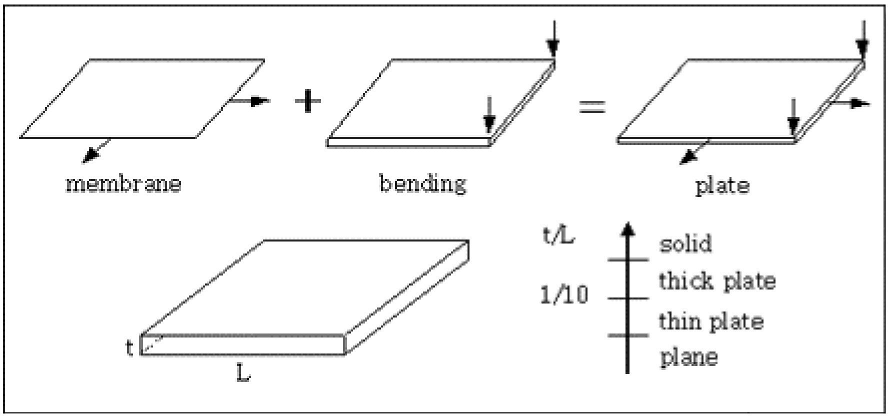
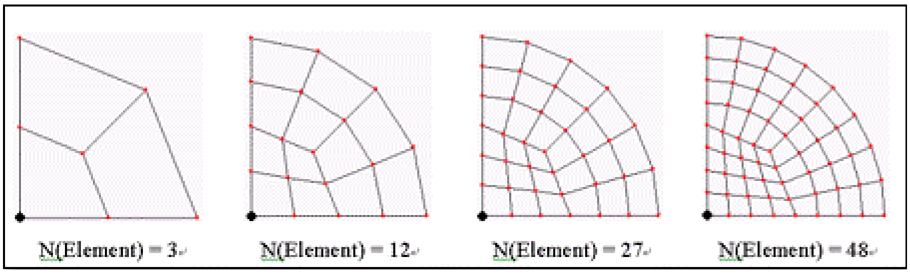


Figure 1. Classification of Plate Elements

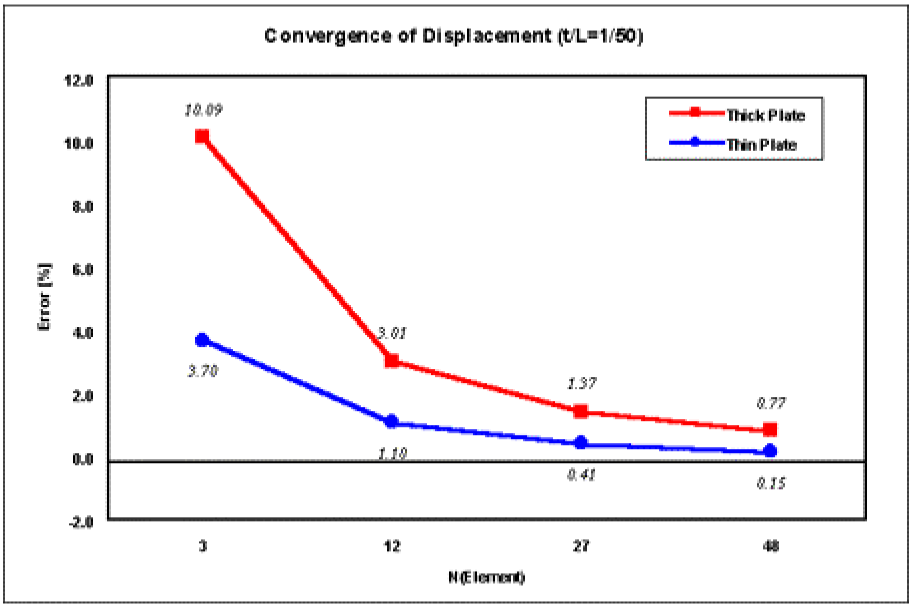
Plate elements are classified in thin and thick plates depending on the thickness-to-length ratio, as shown in Fig.1. If the plate thickness is relatively small (thickness-to-length ratio < 1/10) then the plate is considered thin and the effect of shear deformations can be ignored. In this case the bending stiffness becomes dominant. If the ratio of thickness-to-length is larger than one-tenth (1/10) the plate is considered thick and both bending and shear deformations must be accounted for.

In a thin plate, the margin of error resulted by neglecting the shear deformation effect is as little as 2%. If the thickness is very small, it is more efficient to use plane stress (membrane) element instead of the plate element. On the contrary, if the ratio is very large, it is desirable to use three-dimensional solid elements. Also, the user can separately input in-plane and out-of-plane thicknesses, where the former is the membrane thickness and the later is the bending thickness. As such, the stiffness for each behavior can be adjusted by the respective thicknesses.

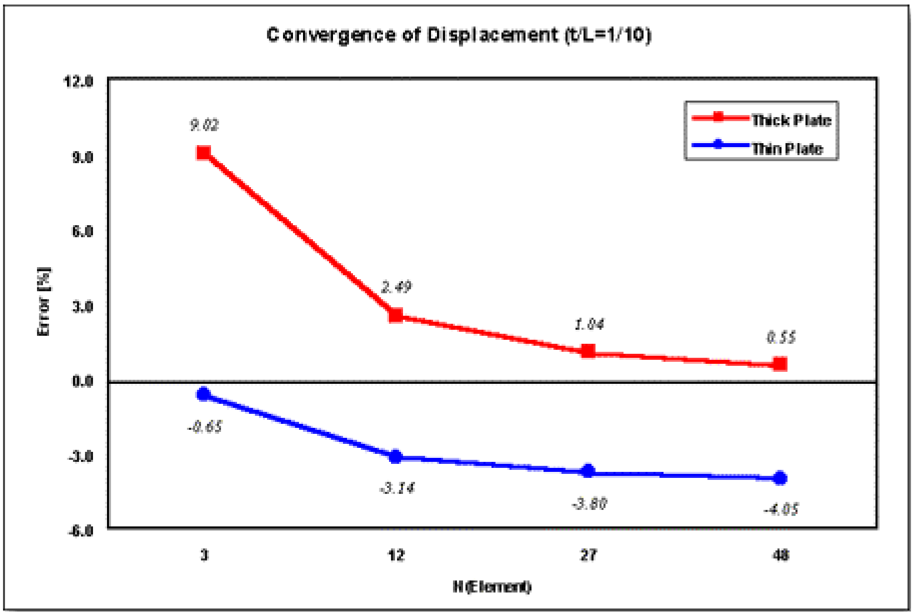
To evaluate the efficiency of the plate elements provided by MIDAS, a typical model (1/4 circular plate) is considered as shown in Fig. 2(a). Figs. 2(b) to (d) illustrate the displacement convergence of thick and thin plates for different thickness-to-length ratios and element meshes. The positive and negative signs of the errors represent that the calculated value is larger or smaller than the theoretical value respectively.



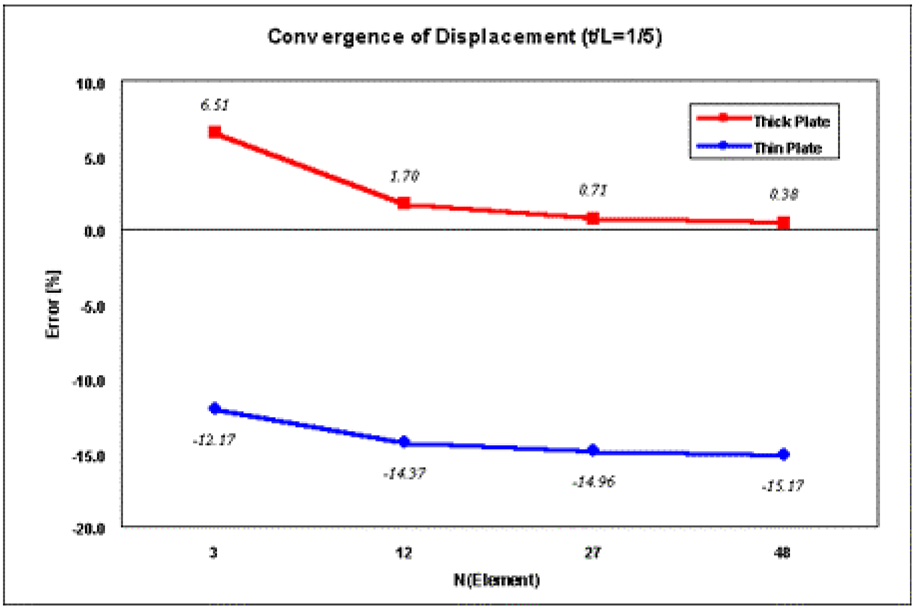
a) Adopted Element Meshes and Location of Measured Displacement for Test Model



b) Displacement Convergence for $t/L = 1/50$



c) Displacement Convergence for $t/L = 1/10$



d) Displacement Convergence for $t/L = 1/5$

Figure 2. Displacement Convergences for Thick Plate and Thin Plate

As shown in Figs. 2(b) and (c), if the thickness-to-length ratio is equal to or less than one-tenth (1/10), the thin plate element produces more accurate results than the thick plate. However, the thick element produces better results if the ratio is larger than 1/10. In the case of the thick plate (t/L=1/5), we can observe that the errors associated with the thin plate element are much larger than those produced by the thick plate element, regardless of the mesh density

Shape Factors Affecting the Performance of Plate Elements

The major element shape factor, which has a great impact on the analysis results, is the Jacobian Determinant, which affects the element stiffness performance. Other shape factors such as aspect ratio, skew angle, taper, warping, etc. are the factors, which affect to a large extent the interpolation of various results at any point within the element domain based on the computed nodal displacements.

If strains and stresses at a particular point cannot be properly calculated based on the computed nodal displacements, it is referred to as ‘interpolation failure’. This is closely related to locking phenomenon whereby the structure’s behavior suddenly becomes stiff, as a result of excessive stiffness calculated for a particular deformation state. Such a locking phenomenon will likely occur when the shape factors such as aspect ratio, skew angle, taper and warping are very poor.

(1) Jacobian Determinant $\left|\mathbf{J}\right|$

For an isoparametric element, the stiffness is calculated by numerical integration using natural coordinates. Transformation into natural coordinates is illustrated in Fig. 3. After transformation, the arbitrarily shaped element is mapped into a square master element with the length of 2. Here, Jacobian determinant $\left|\mathrm{J}\right|$ denotes the ratio of the original element's area to the master element's area (see Fig. 3).

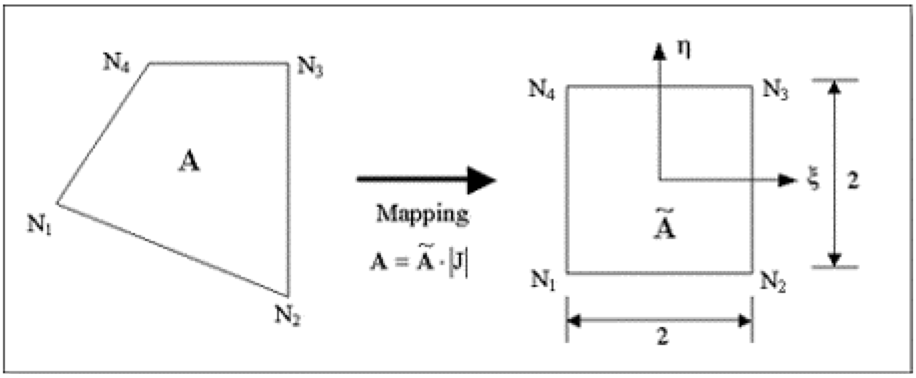


Figure 3. Mapping of Isoparametric Element to Square Element

It should be noted that the shape of the four-node quadrilateral element must be of convex form. In other words, the element sides should not intersect each other (see Fig. 4). If the configuration of the quadrilateral element is not convex, the value of $\left|\mathrm{J}\right|$ is 0 or negative, and the element stiffness has a 0 or negative value. As a reference, the ratio of *maximum* and *minimum* values of Jacobian determinant should satisfy the condition $\left|\mathbf{J}\right|_{max} / \left|\mathbf{J}\right|_{min} \leq 2$. The farther the interior angles in a four-node quadrilateral element are from 90°, the larger the value.

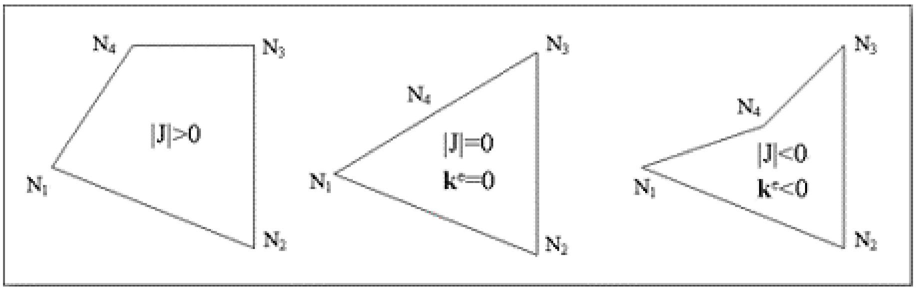


Figure 4. Relation between Element Shape and Jacobian Determinant

(2) Aspect Ratio Λ

Aspect ratio as shown in Fig. 5 is the length ratio of the shortest side to the longest side. The best element shape is a square whose aspect ratio is 1, and the aspect ratio becomes much smaller than 1 for a poorer shape. Therefore, when stress evaluation is important the aspect ratio should not be over one-third, and when the deformation (displacement) evaluation is important it should not be over one-fifth. It should be noted that the results of nonlinear analysis are more sensitive to the change of aspect ratio compared to the results of linear analysis.

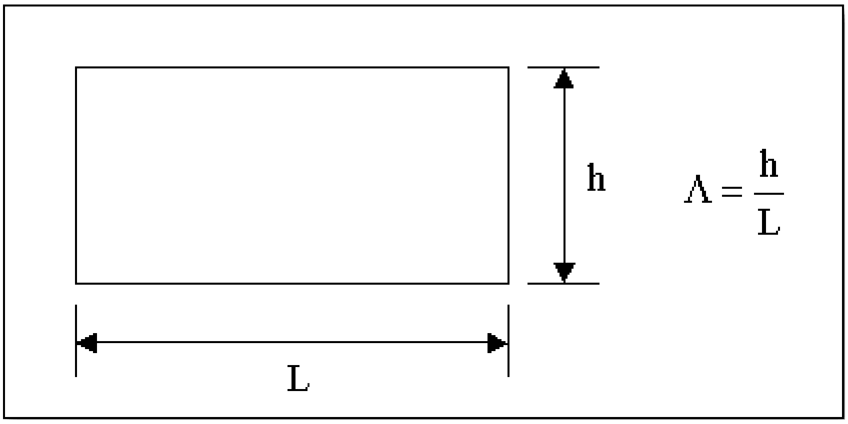


Figure 5. Aspect Ratio

(3) Skew Angle α

Skew angle denotes the degree of angular deviation from a rectangle. The best element shape is a square whose skew angle is 0, and the skew angle is farther away from 0 for a poorer shape. To obtain accurate analysis results, it is recommended that the skew angle be maintained no less than 45°, and all the interior angles of the quadrilateral element should remain in between 45° and 135°.

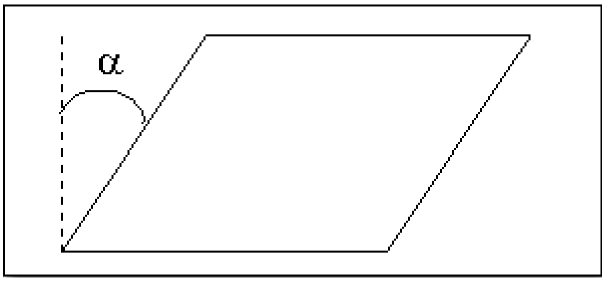
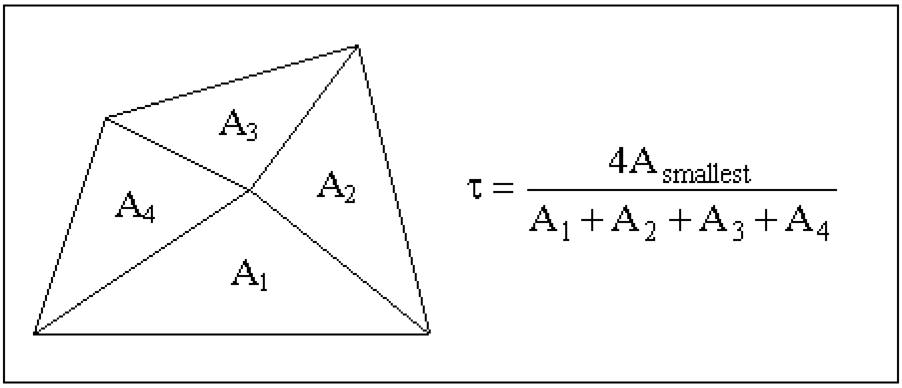


Figure 6. Skew Angle

(4) Taper τ

Taper represents the measure of geometric deviation of the element form from a rectangle. Accordingly, the best value of taper is equal to 1. The value of taper farther from 1 indicates a poor element shape

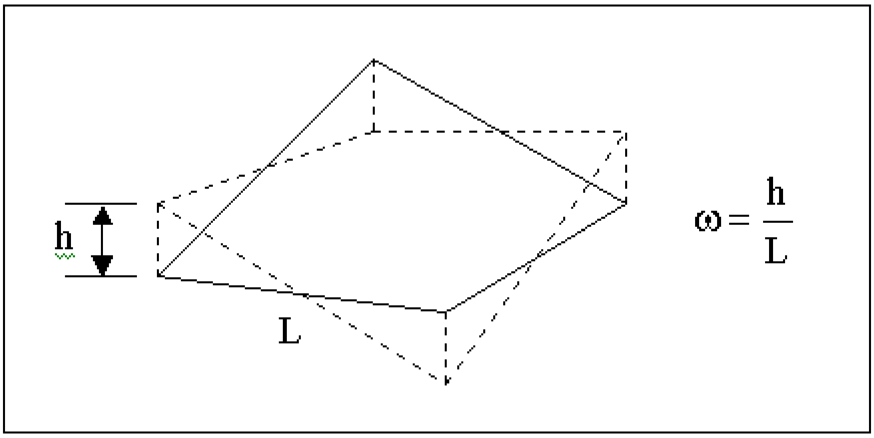


$$\tau = \frac{4A_{smallest}}{A_1 + A_2 + A_3 + A_4}$$

Figure 7. Taper

(5) Warping ω

Warping measures the degree of deviation (plane distortion) of four nodes from a plane. While aspect ratio, skew angle and taper evaluate in-plane offset, warping evaluates out-of-plane offset. The magnitude of warping should not exceed one-hundredth. Especially, it is necessary to be cautious with quadrilateral elements created in the vicinity of the intersections (connections) of curved surfaces.



$$\omega = \frac{h}{L}$$

Figure 8. Warping

■ Items to be considered when Using Plate Elements

- For any element shape, the use of quadrilateral elements is better than triangular elements. However, when the quadrilateral shape deviates from the permitted limit range, the use of triangular elements is preferable.

- Since bending stresses exist in plate elements, the top and the bottom surfaces are distinguished. Therefore, the normal direction of the adjacent plate elements must be coincident in order to align the top and the bottom surfaces of the contiguous elements.

- Based on the nodal degrees of freedom (DOF), elements are classified as elasticity elements with nodal DOFs, which include only translational displacements and structural elements with nodal DOFs, which include translational and rotational displacements. Hinges are formed at the nodes where these two types of elements meet. The plate element can be used in combination with three-dimensional element (solid element). In such a case, moment should be transmitted at the connections of the elements by including additional plate elements shared by both types of elements or using rigid links or rigid beam elements.

- The plate element does not have the rotational stiffness about normal direction (drilling). Therefore, it is necessary to restrain the drilling degree of freedom (RZ) or to put a fictitious rotational spring with small stiffness to avoid singularity error. If a beam is connected perpendicular to the plate element without the drilling DOF, an additional beam element should be introduced within the plate element to transmit torque. MIDAS automatically assigns fictitious springs with appropriate stiffness to the plate elements.

- If triangular elements are used in a symmetric structure, the mesh layout should be also kept symmetrical in order to obtain symmetrical analysis results. Note that MIDAS contains a function, which produces the total results based on a ½ or ¼ model analysis.