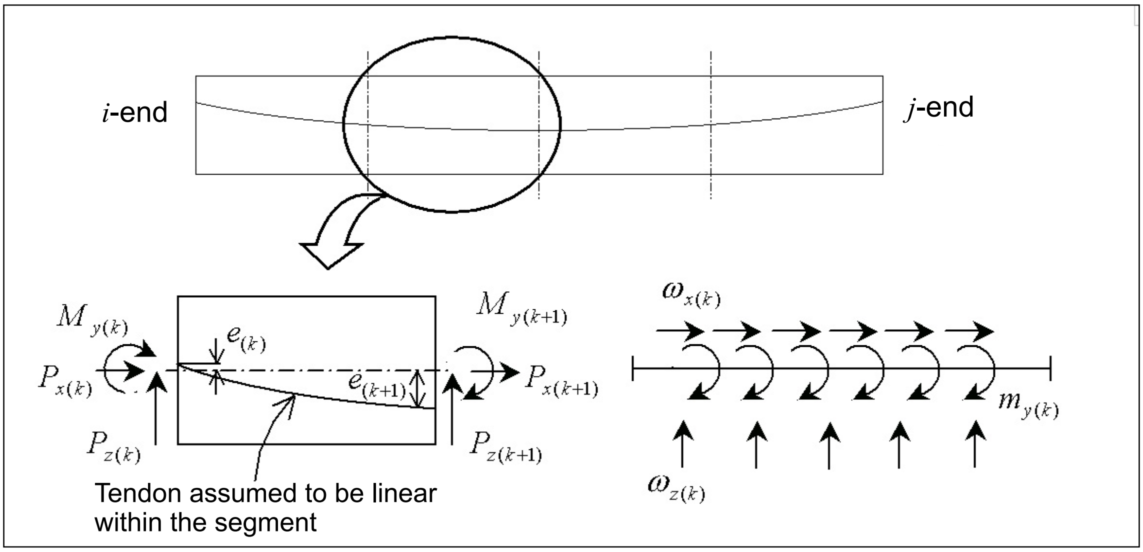
Formulation of Equivalent Beam Load due to Prestress

In finite element analysis, the induction of tendon prestress is represented by the application of load into the element. To accomplish this, it is necessary that a beam load, equivalent to the action of the prestress, be generated and applied to the element. It is however impractical to create an equivalent beam load, which is completely coinciding with the prestress. This is because that the load varies along the length of the beam relative to the tendon profile and the characteristics of prestress loss. Even if we attempt to cover all the local effects, too many beam loads will deteriorate the efficiency of calculations.

MIDAS thus recognizes the issue and adopts the method of segmenting the beam in 4 equal lengths to best approximate the equivalent beam loads in the segments. The following equations best summarize this approach. The basic concept of deriving the equations is founded on the fact that the tendon prestress must satisfy its self-equilibrium. For the sake of simplicity, we now assume that the tendon is located in the x-z plane in the k-th segment.



$$P_{x(k)} = P\cos\theta_{(k)} \qquad P_{x(k+1)} = \left(-P\right)\cos\theta_{(k)} = -P\cos\theta_{(k)}$$

$$P_{z(k)} = P\left(-\sin\theta_{(k)}\right) = -P\sin\theta_{(k)} \qquad P_{z(k+1)} = \left(-P\right)\left(-\sin\theta_{(k)}\right) = P\sin\theta_{(k)}$$

$$M_{y(k)} = P_{x(k)}e_{(k)} \qquad M_{y(k+1)} = P_{x(k+1)}e_{(k+1)}$$

$$\sum F_x = P_{x(k)} + w_{x(k)}\frac{l}{4} + P_{x(k+1)} = 0$$

$$\sum F_z = P_{z(k)} + w_{z(k)}\frac{l}{4} + P_{z(k+1)} = 0$$

$$\sum M_{y(k+1)} = M_{y(k)} + P_{z(k)}\frac{l}{4} + w_{z(k)}\frac{l}{4}\frac{l/4}{2} + m_{y(k)}\frac{l}{4} + M_{y(k+1)} = 0$$

$$w_{x(k)} = -\frac{P_{x(k)} + P_{x(k+1)}}{l/4}$$

$$w_{z(k)} = -\frac{P_{z(k)} + P_{z(k+1)}}{l/4}$$

$$m_{y(k)} = -\frac{M_{y(k)} + M_{y(k+1)}}{l/4} - P_{z(k)} - w_{z(k)}\frac{l/4}{2}$$

where, $P_{x(k)}, P_{z(k)}, M_{y(k)}$: Loads in x & z-directions and moment about y-axis respectively at the start of the k-th segment

$w_{x(k)}, w_{z(k)}, m_{y(k)}$: Distributed loads in x & z-directions and moment about y-axis respectively at the k-th segment

$\cos\theta_{(k)}, \sin\theta_{(k)}$: Angles of tendon inclination relative to the x-axis at the k-th segment