Algorithm for calculating the loss of tendon Prestress

The loss of tendon prestress is classified into three kinds; namely, the loss during seating, the instant loss after seating and the long-term loss.

During the seating operation the loss by instant elastic deformation of the concrete occurs due to the applied tension force. MIDAS however considers that this shortening effect has been accounted for by the prestress load. Accordingly, this loss is excluded from the calculation of prestress loss.

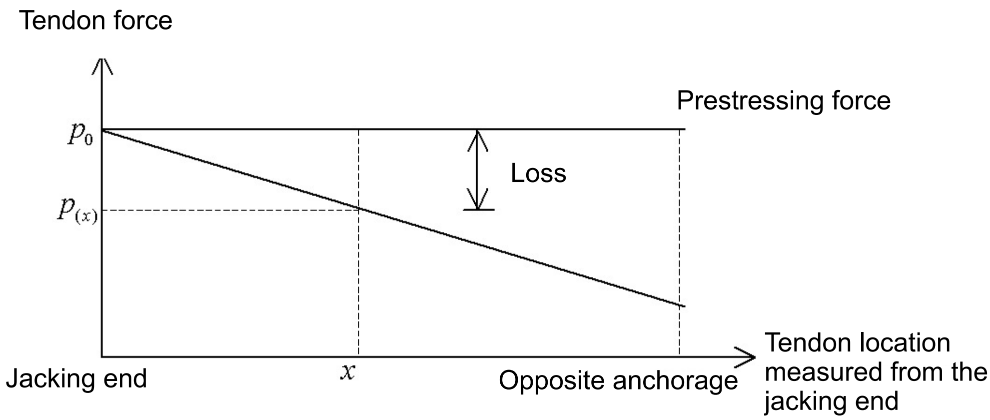
Prestress losses immediately after seating include friction loss and the loss due to anchorage slip. Friction loss is divided into the curvature friction and wobble friction. The expression for calculating the friction losses is as follows:

$$P(x) = P_0 e^{-(\mu\theta + kx)}$$

where, P_0: magnitude of prestress, x: distance from the jacking end,

θ : Accumulated angle, μ : Curvature coefficient, k: Wobble coefficient

The friction loss will not be considered for pre-tensioned tendons and external post-tensioned tendons.



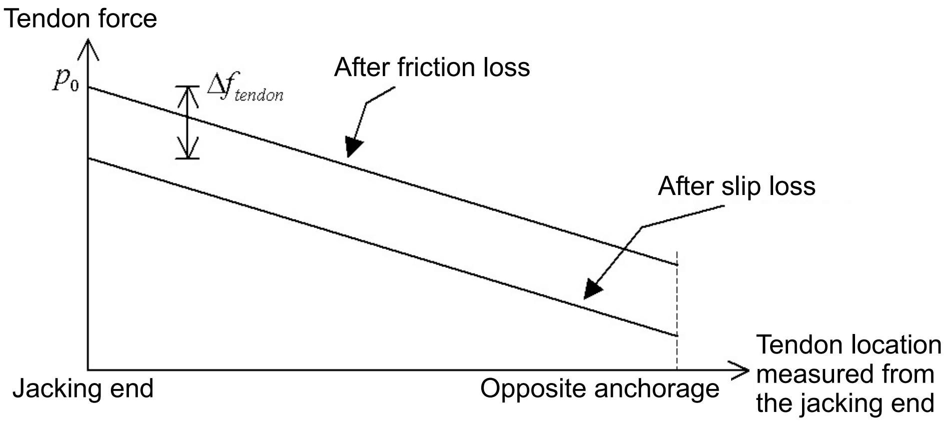
Friction Loss due to tensioning from one end

After calculating the friction loss using the above equation, the loss due to the seating slip is calculated. Two methods can be used to calculate the loss due the seating slip. The first method directly calculates the loss due to the change in tendon length neglecting the reverse friction and the second method calculates the precise loss including the reverse friction. MIDAS adopts the former method to calculate the slip loss, as it neglects the friction in the cases of pre-tensioned and external post-tensioned tendons. On the other hand, it calculates the exact loss in the case of internal post-tensioned tendons considering the reverse friction.

Where the reverse friction is neglected, the following equation is used to calculate the loss:

$$\Delta f_{\text{tendon}} = E_{\text{tendon}} \frac{\Delta l}{L}$$

where, Δf_{tendon} : Magnitude of prestress loss in tendon, E_{tendon}: Modulus of elasticity of tendon,
Δl : Magnitude of slip at seating, L: Total tendon length



Case where reverse friction is neglected

Tension loss calculation becomes a little bit complicated if the reverse friction is accounted for. The basic concept is founded on equalizing the reduction of strain energy due to the slip and the generation of friction energy due to the reverse friction. If we assume that the friction coefficient is uniform throughout the entire tendon length, we can express the following equation:

$$\underbrace{E_{\text{tendon}} A_{\text{tendon}} \Delta l}_{\text{slip energy}} = \underbrace{\frac{1}{2} \Delta f_{\text{tendon}} l_{set}}_{\text{friction energy}}$$

$$\Delta f_{\text{tendon}} = 2 s l_{set}$$

$$l_{set} = \sqrt{\frac{\Delta l E_{\text{tendon}} A_{\text{tendon}}}{s}}$$

where, A_{tendon}: Area of tendon, l_{set}: Length of tendon undergoing slips,

s: Friction coefficient Others as before

Generally, the friction coefficient varies along the tendon. The basic concept for calculating the loss remains unchanged in this case as well, but we will resort to segmenting the tendon for varying friction coefficients. If we assume that there are N segments for different friction coefficients, we need to examine which of the segments that l_{set} is occurring. If we assume that l_{set} lies within the n-th segment, then the following is observed:

$$\triangle f_1 = 2 s_1 L_1 ,\ \ \triangle f_2 = 2 s_2 \left(L_2 - L_1 \right),\ \ldots,\ \ \triangle f_n = 2 s_n \left(l_{set} - L_{n-1} \right)$$

$$\triangle f_{\text{tendon}} = \sum_{i=1}^{n} \triangle f_i$$

$$\begin{aligned} E_{\text{friction}} &= \frac{1}{2} \triangle f_1 L_1 + \left[\triangle f_2 L_1 + \frac{1}{2} \triangle f_2 \left(L_2 - L_1 \right) \right] + \ldots + \frac{1}{2} \triangle f_n \left(l_{set} - L_{n-1} \right) \\ &= \frac{1}{2} \triangle f_1 L_1 + \sum_{i=2}^{n-1} \left[\frac{1}{2} \triangle f_i \left(L_i - L_{i-1} \right) + \triangle f_i L_{i-1} \right] + \triangle f_n L_{n-1} + \frac{1}{2} \triangle f_n \left(l_{set} - L_{n-1} \right) \\ &= \mathrm{C} + \triangle f_n L_{n-1} + \frac{1}{2} \triangle f_n \left(l_{set} - L_{n-1} \right) \end{aligned}$$

$$E_{\text{tendon}} A_{\text{tendon}} \triangle l = E_{\text{friction}}$$

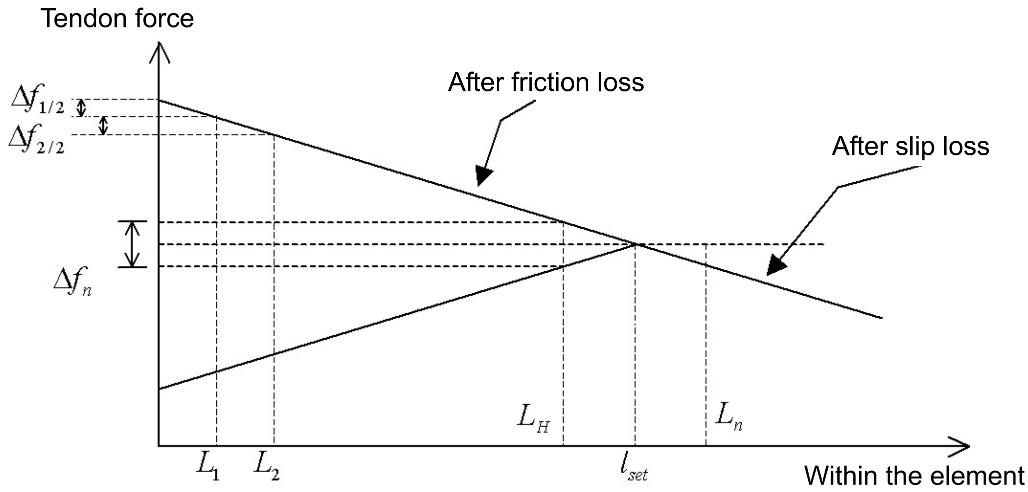
$$s_n \left(l_{set} - L_{n-1} \right)^2 + 2 s_n L_{n-1} \left(l_{set} - L_{n-1} \right) + \mathrm{C} - E_{\text{tendon}} A_{\text{tendon}} \triangle l = 0$$

$$l_{set} = L_{n-1} + \frac{-s_n L_{n-1} + \sqrt{s_n^2 L_{n-1}^2 - s_n \left(\mathrm{C} - E_{\text{tendon}} A_{\text{tendon}} \triangle l \right)}}{s_n} = \sqrt{L_{n-1}^2 - \frac{\mathrm{C} - E_{\text{tendon}} A_{\text{tendon}} \triangle l}{s_n}}$$

where, Δf_i: Loss in i[th] segment s_i: Friction coefficient in i[th] segment,

L_i: Accumulated length to i[th] segment, C: Constant to simplify the expressions

From the above expression, if l_{set} is between L_{n-1} and L_n, the assumption is correct, and the calculated Δf_{tendon} represents an accurate loss.



Four types of long term prestress losses can be identified, which are due to creep, shrinkage, elastic deformation and stress relaxation. The first three types relate to the direct change in tendon length due to the member deformation. There are several expressions for calculating creep and shrinkage. MIDAS mainly supports the expressions specified in the design standards and the user-defined calculations. Elastic deformation on the other hand is calculated by element displacements due to member forces, and it does not require special transformations.

On the other hand, the tension loss due to stress relaxation does not accompany the change in tendon length. That is, when steel is subjected to stress exceeding a certain level, the stress decreases with time without incurring any other changes. The expression for calculating relaxation is as follows:

$$\triangle\sigma_{r1} = \sigma_{si0}\left[1 - \frac{\log(t_1 - t_0)}{10}\left(\frac{\sigma_{si0}}{f_y} - 0.55\right)\right], \quad \left(\frac{\sigma_{si0}}{f_y} \geq 0.55\right)$$

$$\sigma_{sik} = \frac{\left(1 + \frac{0.55\log(t_k - t_0)}{10}\right) - \sqrt{\left(1 + \frac{0.55\log(t_k - t_0)}{10}\right)^2 - 4\frac{\log(t_k - t_0)}{10 f_y}\sigma_{sk}}}{2\frac{\log(t_k - t_0)}{10 f_y}}, \quad (k \geq 1)$$

$$\triangle\sigma_{r(k+1)} = \sigma_{sik}\,\frac{\log\left(t_{k+1}-t_0\right)-\log\left(t_k-t_0\right)}{10}\left(\frac{\sigma_{sik}}{f_y}-0.55\right),\quad \left(k\geq 1,\ \frac{\sigma_{sik}}{f_y}\geq 0.55\right)$$

$$\sigma_{n,\text{loss}} = \sum_{k=1}^{n}\Delta\sigma_{rk}$$

where $\Delta\sigma_{rk}$: Stress loss within the time interval k, t_k: Time elapsed until the time interval k,

σ_{sk} : Stress before loss within the time interval k,

σ_{sik} : Initial stress at t_0 to become σ_{sk} within the interval k

$\sigma_{n,\text{loss}}$: Total loss of stress until the time interval n

Tendon relaxation reduces the tension in the entire tendon and the change in tension along the length as well.